CONTROL OF PORT HAMILTONIAN SYSTEMS BY DISSIPATIVE DEVICES AND ITS APPLICATION TO IMPROVE THE SEMI-ACTIVE SUSPENSION BEHAVIOUR

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Abstract: The port Hamiltonian framework is a powerful tool for modeling a wide class of nonlinear systems such as robots and, more generally, mechatronic systems. A wide variety of mechatronic systems are controlled by operating dissipative components and the standard approaches for the control of port Hamiltonian systems are not applicable. Facing the limitation that the controlled devices can only dissipate power, the issue is to find a proper control law to satisfy the control requirements. This paper proposes to choose the control inputs to lead the input power of a subsystem in oder to satisfy the requirements by controlling the energy stored or the power dissipated in that subsystem. A slight extension of the definition of port Hamiltonian system is proposed to allow the description of a larger set of mechatronic systems. Although some important issues remain open, the example of the semi-active suspension shows that some positive results can be achieved by applying the proposed approach. *Copyright* © *2006 IFAC*

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1. INTRODUCTION

From a mathematical perspective, the *port controlled Hamiltonian systems* (PCH) (van der Schaft, 2000), are natural candidates to model many real systems, as shown in the application examples cited in (Ortega *et al.*, 2002). Basically, PCH are systems defined with respect to a geometric structure capturing the basic interconnection and dissipation laws, and a *Hamiltonian* function given by the total stored energy of the system.

The control of PCH is an interesting research topic and outstanding results have already been obtained. The main results presented so far consider the possibility to operate on the power-ports of a system in order to obtain a controlled closed-loop system that is still a PCH with desired Hamiltonian function, interconnection laws and damping, see (Ortega *et al.*, 2002) and the references therein. Another approach is the control by interconnection of PCH described in (Garcia-Canseco *et al.*, 2005). However many mechatronic systems are not controlled by means of the power ports but controlled by operating dissipative components such as variable resistors, variable dampers, clutches, some electro-valves and more. The issue is to find a proper control law that allows to satisfy the control requirements facing the limitation that the energy can only be dissipated by the controlled devices. To the best of our knowledge the problem of controlling a PCH by means of dissipative components has not already been addressed.

The key idea proposed in this paper is to divide the PCH system into two or more PCH subsystems that are connected by a power preserving connection. The control inputs are then chosen to control the power flowing towards a certain subsystems or to control the stored energy in that subsystem. To this aim, a slight extension of the definition of PCH is proposed to allow the description of a larger set of mechatronic systems and to obtain an explicit representation of the power flowing to a subsystem. Thanks to the dissipative nature of the controlled devices the passivity properties of the given PCH system are preserved.

The semi-active vehicle suspensions (Savaresi *et al.*, 2003) are an example of a mechatronic system with a controllable dissipative device. By following the proposed approach, some of the control laws already presented in literature for the semi-active suspensions are derived again and an energetic interpretation is given. Moreover a new control with improved performances is proposed.

The paper is organized as follows: section 2 gives a brief introduction on Hamiltonian systems and extends the definition of PCH. The proposed control law for the dissipative components are presented in section 3. The application example to semi-active suspensions is described in section 4.

2. AN EXTENSION OF THE PORT HAMILTONIAN DEFINITION

The port-Hamiltonian framework is a powerful means to model robotic, mechatronic and dynamic systems. A brief recall of some definitions written in (van der Schaft, 2000) is given herein for reader convenience. The *port-controlled Hamiltonian system* (PCH) are systems of the form:

$$\dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u$$
$$y = g^T(x)\frac{\partial H}{\partial x}(x) \qquad (1)$$
$$J(x) = -J^T(x) \qquad R(x) = R^T(x) \geq 0$$

One of the key feature of the PCH is the energy perspective in modeling the physical systems. The Hamiltonian $H(x)$ represent the energy stored in the system, the product $y^T u$ has the units of power and has the physical meaning of the power flowing through the port (u, y):

$$y^T u = \frac{dH}{dt} + \frac{\partial H}{\partial x}^T R\frac{\partial H}{\partial x} \geq \frac{dH}{dt}$$

namely the power $y^T u$ supplied to the system is partially stored as energy and partially dissipated through R. Many PCH can be obtained connecting different subsystems by power preserving interconnections. Let (u_1, y_1) and (u_2, y_2) be the power ports of two PCH, the general power preserving interconnection is the following:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad (2)$$

the matrix A can also be time varying and/or state dependent. With the interconnection (2) the power flows from one system to the other without losses: $y_1^T u_1 = y_1^T A y_2 = y_2^T A^T y_1 = -y_2^T u_2$, namely the outcoming energy from one subsystem is exactly the incoming energy to the other.

The PCH in (1) does not consider the possibility of external inputs that directly modify either the dissipation matrix $R(x)$ or the matrix $J(x)$. This problem was partly addressed in (Perez *et al.*, 2004) where a matrix J depending on external inputs is considered. As previously described, many mechatronic systems have dissipative components whose behaviour depends on an external input. To represent mechatronic systems as

a set of PCH connected by power preserving interconnections, the definition (1) is not enough, as shown in the example of Sec. 4. Some components of mechatronic systems may show a direct dissipation between the input u and the output y. A resistor is simplest example. The PCH in (1) cannot consider such behaviour since the dissipation is only related to the gradient of $H(x)$. To take into account these phenomena, the following modification of (1) is proposed:

$$\dot{x} = [J_1(x, v) - R_1(x, v)] \frac{\partial H}{\partial x}(x) + g(x, v) u$$
$$y = g^T(x, v) \frac{\partial H}{\partial x}(x) - [J_2(x, v) - R_2(x, v)] u \quad (3)$$
$$J_i(x, v) = -J_i^T(x, v) \qquad i = 1, 2$$
$$R_i(x, v) = R_i^T(x, v) \geq 0 \qquad i = 1, 2$$

where v is an external input vector that may also be equal to u. The matrix $J_2(x, v)$ models a direct change of interconnection (example: ideal switch). The matrix $[J_2(x, v) - R_2(x, v)]$ has a similar meaning as the matrix "D" of the linear systems. This new definition (3) is similar to the one in (Escobar *et al.*, 2004), however in that paper the so called *throughput* matrix relating u and y was defined as a skew-symmetric matrix. Therefore, differently from the definition of the matrix $[J_2(x, v) - R_2(x, v)]$ in (3), the *throughput* matrix is unable to describe a direct dissipation. The extended definition (3) preserves the basic properties of the PCHs and the energy perspective in modeling the physical systems. The inner product $y^T u$ has still the physical meaning of the power flowing through the port (u, y) and the power balance in (3) is the following:

$$\frac{dH}{dt} = y^T u - \frac{\partial H^T}{\partial x} R_1(x, v)\frac{\partial H}{\partial x} - u^T R_2(x, v)u \quad (4)$$

From (4) it is straightforward to verify that (3) satisfies the energy balance equation (EBE):

$$H(x(t)) - H(x(0)) = \int_0^t y^T(\tau)u(\tau)d\tau - D(t) \quad (5)$$

where $D(t)$ is a nonnegative function that captures the dissipation effects.

3. CONTROL BY DISSIPATIVE COMPONENTS

Many mechatronic systems are controlled by dissipative components and the inputs u in (3) are not controlled variables, conversely the inputs u often represent disturbances. The semi-active suspension described in the next section is a such example: the input u is the road profile velocity $\dot{x}_r$. Further examples are the clutches (the torques on the axles are not controlled inputs, only the friction torque is controlled) and some electro-valves (the main external inputs are usually the hydraulic supply pressure and the reservoir pressure). If it is not possible to modify the power flows by the power port (u, y), the approaches (Ortega *et al.*, 2002) and (Garcia-Canseco *et al.*, 2005) cannot be used. The control requirements can be

satisfied by operating the dissipative components only. This control problem, to the best of our knowledge, has never been addressed for PCH and a full result is not yet available. This paper proposes to control the dissipative components by taking into account the power exchanges between subsystems. This approach is based on two steps:

1) translate the control requirements in a required energy level for a subsystem or in a required input power to a subsystem;
2) operate the dissipative components to obtain the desired energy level or input power.

To help the solution of the first part, the mechatronic system is divided into two or more subsystems of the type (1) or (3) that are connected by a power preserving connection of the type (2). By this way the input power and the energy stored in each subsystem can be easily computed. The correspondence between control requirements and energy levels or input power is the target of future research and it is not addressed in this paper.

Concerning the second part, the control inputs v are chosen to control the input power or the energy stored in a subsystem. Four control laws for the input v are proposed. The control, to a desired value W_d, of the input power of a subsystem is the target of the control laws C1 and C2. The control laws C3 and C4 are based on a desired energy level H_d for the subsystem. To simplify the notation, let $d(v, x)$ denote the positive dissipated power:

$$d(v, x) = \frac{\partial H^T}{\partial x} R_1(x, v) \frac{\partial H}{\partial x} + u^T R_2(x, v) u$$
$$\dot{H} = y^T u - d(v, x) \tag{6}$$

C1) Let W_d be the desired value of the input power of a subsystem of the type (1) or (3). From the power balance (6), the desired power W_d or its closest possible value is obtained by choosing the control input v as follows:

$$v : \begin{cases} \text{argmax}(\dot{H} + d) \text{ if argmax}(\dot{H} + d) < W_d \\ \text{argmin}(\dot{H} + d) \text{ if argmin}(\dot{H} + d) > W_d \\ W_d = \dot{H} + d \qquad \text{otherwise} \end{cases} \tag{7}$$

The first (or the second) condition means that the desired power W_d cannot be obtained and then one of the values of v that gives the smallest difference is chosen. The desired value W_d is only obtained in the third case.

C2) When it is required to minimize or maximize the power W_d the control law (7) can be simplified and the resulting new control law may be much easier to be implemented. The minimization (maximization) of W_d can be obtained by substituting in (7) $W_d = -\infty$ ($W_d = \infty$). If $W_d = \pm\infty$ only some conditions of (7) are possible and the control (7) simplifies as follows:

$$v : \begin{cases} \text{argmax}(\dot{H} + d(v, x)) \text{ if } W_d = +\infty \\ \text{argmin}(\dot{H} + d(v, x)) \text{ if } W_d = -\infty \end{cases} \tag{8}$$

As shown in the next section, this control law may be much simpler to be implemented since only the

maximization (or minimization) of $\dot{H}(x) + d(v, x)$ with respect to v is required.

C3) Let H_d be the desired level of energy for the considered subsystem. Let $f(z)$ be a function of the variable z such that $f(z) z > 0$ if $z \neq 0$. The desired energy level H_d can be obtained from:

$$\dot{H}_d = -f(H(x) - H_d) \tag{9}$$

where $\dot{H}_d$ is the desired value for the time-derivative $\dot{H}(x)$. From (4) the time derivative $\dot{H}_d$ can be obtained choosing the control v as follows:

$$v : \begin{cases} \text{argmax}(y^T u - d) \text{ if argmax}(y^T u - d) < \dot{H}_d \\ \text{argmin}(y^T u - d) \text{ if argmin}(y^T u - d) > \dot{H}_d \\ \dot{H}_d = y^T u - d \qquad \text{otherwise} \end{cases}$$
$$\tag{10}$$

The first (or the second) condition means that the desired $\dot{H}_d$ cannot be obtained and then one of the values of v that gives the smallest difference is chosen. The desired value $\dot{H}_d$ is only obtained in the third case.

C4) A control law that requires less knowledge of the system than C3 is obtained starting from (9) and (10) and choosing the maximum or the minimum possible values for $\dot{H}_d$. The resulting control is the following:

$$v : \begin{cases} \text{argmax}(y^T u - d(v, x)) \text{ if } H(x) < H_d \\ \text{argmin}(y^T u - d(v, x)) \text{ if } H(x) \geq H_d \end{cases} \tag{11}$$

As shown in the next section, this control law may be much simpler to be implemented since only the maximization (or minimization) of $y^T u - d(v, x)$ with respect to v is required.

Remark 1. As shown in equations (7) and (10) it is not ensured that the control requirements can always be satisfied by operating on the control input v. This is mainly due to the inputs u that are not controlled variables and that may assume any value while the term $d(v, x)$ may be limited.

Remark 2. In the more general case the input v is a vector, therefore equations (7), (8) (10) and (11) may have more solutions that have different components of v. If a particular system structure is not given, it will not be possible to define a criterion for the choice of the best solution.

4. CONTROL OF SEMI-ACTIVE SUSPENSIONS

The semi-active suspensions are a typical example of a mechatronic system controlled by a dissipative component. A semi active suspension system is shown in Fig. 1 regarding a quarter-car model. The damping b of the shock absorber is controlled by an electro-valve. The typical control problem is to choose the value of the desired damping b_d in order to maximize the comfort for the passengers. The ideal solution were to obtain a body (sprung mass) speed and acceleration as close as possible to zero to minimize the movements and the forces perceived by the passengers. A detailed description of the semi-active suspensions can be found
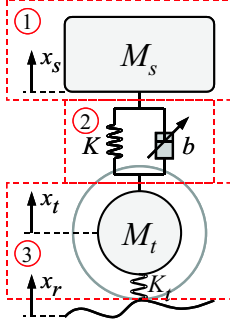
Fig. 1. Graphical representation of a quarter-car model with semi-active suspension.

in (Savaresi *et al.*, 2003) and in the references therein. This section shows how some control laws already known for the semi-active suspensions can be derived again by means of the proposed approach. Moreover a new control with slightly better performances is proposed.

The variables shown in Fig. 1 have the following meanings: M_s denote the quarter-car body mass, M_t is the total unsprung mass (tire, wheel, brakes, suspension links,...), b and b_d are the real and the desired damping coefficients of the shock-absorber, K and K_t are the stiffness of the suspension spring and of the tire, respectively. Finally x_s, x_t and x_r are the vertical position of the body mass, of the tire and of the road profile, respectively. The PCH model of the system shown in Fig. 1 is the following:

$$H = \frac{1}{2}M_s \dot{x}_s^2 + \frac{1}{2}M_t \dot{x}_t^2 + \frac{1}{2}K x_{st}^2 + \frac{1}{2}K_t x_{tr}^2$$

$$\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_t \\ \dot{x}_{st} \\ \dot{x}_{tr} \end{bmatrix} = \begin{bmatrix} \frac{-b}{M_s^2} & \frac{b}{M_s M_t} & \frac{-1}{M_s} & 0 \\ \frac{b}{M_s M_t} & \frac{-b}{M_t^2} & \frac{1}{M_t} & \frac{-1}{M_t} \\ \frac{1}{M_s} & \frac{-1}{M_t} & 0 & 0 \\ 0 & \frac{1}{M_t} & 0 & 0 \end{bmatrix} \begin{bmatrix} M_s \dot{x}_s \\ M_t \dot{x}_t \\ K x_{st} \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \dot{x}_r$$

$$y = [0\ 0\ 0\ -1]\, [M_s \dot{x}_s\ M_t \dot{x}_t\ K x_{st}\ K_t x_{tr}]^T$$

(12)

The gravitational force has been compensated by the springs pre-load and it does not compare in the equations. The state variables $x_{st} = x_s - x_t$ and $x_{tr} = x_t - x_r$ represent, respectively, the deformations of the spring and of the tire with respect to the equilibrium length.

The variable dissipation b depends on the actuator dynamics. The simplest actuator is usually described by a first order linear dynamics with saturation of b between $b_{min} > 0$ and $b_{max} > b_{min}$. Let b_d be the desired damping, let $\beta > 0$ be the bandwidth of the actuator, the simplified actuator dynamics is the following:

$$\dot{b} = \begin{cases} 0 & \text{if} & b \geq b_{max} \text{ and } b_d \geq b_{max} \\ 0 & \text{if} & b = b_{min} \text{ and } b_d \leq b_{min} \\ \beta\,(b_d - b) & \text{else} \end{cases}$$

Remark 3. For the sake of clarity the described suspension system is linear as in (Savaresi *et al.*, 2003). For a real suspension system both

K and b are nonlinear functions of some state variables. However the results presented in the following section hold also in the nonlinear case.

4.1 Partition of the PCH

The semi active suspension system can be partitioned in the following three connected PCHs (dashed boxes of Fig. 1):
1) Subsystem 1, sprung mass PCH:

$$H_1 = \frac{1}{2} M_s \dot{x}_s^2$$

$$\ddot{x}_s = [\,0\,]\, M_s \dot{x}_s + \begin{bmatrix} \frac{1}{M_s} \end{bmatrix} u_1$$

$$y_1 = \begin{bmatrix} \frac{1}{M_s} \end{bmatrix} \frac{\partial H_1}{\partial \dot{x}_s} = \dot{x}_s$$

(13)

2) Subsystem 2, spring-damper PCH:

$$H_2 = \frac{1}{2} K x_{st}^2$$

$$\dot{x}_{st} = [\,0\,]\, K x_{st} + [\,1\ -1\,] \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix}$$

$$\begin{bmatrix} y_{2,1} \\ y_{2,2} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} K x_{st} + \begin{bmatrix} b & -b \\ -b & b \end{bmatrix} \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix}$$

(14)

3) Subsystem 3, wheel and tire PCH:

$$H_3 = \frac{1}{2} M_t \dot{x}_t^2 + \frac{1}{2} K_t x_{tr}^2$$

$$\begin{bmatrix} \ddot{x}_t \\ \dot{x}_{tr} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{M_t} \\ \frac{1}{M_t} & 0 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_{3,1} \\ \dot{x}_r \end{bmatrix}$$

$$\begin{bmatrix} y_{3,1} \\ y_{3,2} \end{bmatrix} = \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ -K_t x_{tr} \end{bmatrix}$$

(15)

The three subsystems are connected in the following power-preserving way:

$$\begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,1} \\ y_1 \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_s \\ -K x_{st} - b \dot{x}_{st} \end{bmatrix}$$

$$\begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,2} \\ y_{3,1} \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ K x_{st} + b \dot{x}_{st} \end{bmatrix}$$

(16)

4.2 Passive suspensions

Although the passive suspensions are not controllable, their behavior from a power/energy perspective is analyzed to get some insight about the proposed approach. Let consider the spring-damper subsystem 2, the stored energy is:

$$H_2 = \frac{1}{2} K x_{st}^2 \geq 0$$

and the power $y_2^T u_2 = \dot{H}_2 + d_2(b, x)$ results:

$$y_2^T u_2 = \dot{H}_2 + d_2(b, x) = K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2 \quad (17)$$

If the requirement were to dissipate as much power as possible from the external world, the control

law C2 with $W_{2d} = +\infty$ should be applied. From (8) and (17):

$$b : \operatorname{argmax}(K\,x_{st}\,\dot{x}_{st} + b\,\dot{x}_{st}^2) \;\to\; b_d = b_{max}$$

therefore the damping should be constant at its maximum value and a passive suspension is enough to meet the requirement. However it is well known that this solution is not optimal both for handling and for comfort.

4.3 Classic two-state sky-hook control

The target of the sky-hook control is to keep the body vertical speed $\dot{x}_s$ and acceleration $\ddot{x}_s$ as close as possible to zero facing the road profile $\dot{x}_r$. The classic two-state "sky-hook" control law given in the literature, see (Savaresi *et al.*, 2003), is:

$$b_d = \begin{cases} b_{max} & \text{if} \quad \dot{x}_{st}\,\dot{x}_s \geq 0 \\ b_{min} & \text{else} \end{cases} \qquad (18)$$

Consider the subsystem 1 described by (13), the kinetic energy of the body is $H_1(\dot{x}_s)$ and it is always positive. Let the desired energy level H_{1d} be set to zero, this means zero vertical speed of the body. By applying the control law C4, only the second condition of (11) is possible, since for the subsystem 1 $d_1(b, x) = 0$, we have:

$$y_1^T\,u_1 - d_1(b, x) = -b\,\dot{x}_s\dot{x}_{st} - K x_{st}\,\dot{x}_s$$

To minimize $y_1^T\,u_1 - d_1(b, x)$ as requested in (8) it is only possible to minimize $-b\,\dot{x}_s\dot{x}_{st}$, this is obtained exactly by requiring the damping b_d as in the control law (18).

4.4 Acceleration-Driven-Damper control

The Acceleration-Driven-Damper (ADD) control is proposed in (Savaresi *et al.*, 2003) and, under mild assumptions, it is demonstrated to be optimal in the sense that it minimizes the vertical body acceleration $\ddot{x}_s$ when no road-preview is available. The ADD control is defined as:

$$b_d = \begin{cases} b_{max} & \text{if} \quad \ddot{x}_s\,\dot{x}_{st} \geq 0 \\ b_{min} & \text{else} \end{cases} \qquad (19)$$

This control law can be obtained in an alternative way by means of the control law C2 previously presented. Consider the subsystem 2 described by (14). Let the desired power W_{2d} be the following:

$$W_{2d} = \begin{cases} +\infty & \text{if} \quad y_2^T\,u_2 \leq 0 \\ -\infty & \text{if} \quad y_2^T\,u_2 > 0 \end{cases} \qquad (20)$$

The incoming power is $y_2^T\,u_2$ and $W_d = y_{2d}^T\,u_{2d}$ consequently the requirements are to keep the incoming power as close as possible to zero: if $y_2^T\,u_2 > 0$ ($y_2^T\,u_2 < 0$) the requested power W_{2d} is the lowest (highest) possible. This control law mimics a sort of sliding mode control of the power. According to (8), (17) and (20) the damping b must be chosen to maximize (or minimize) $K\,x_{st}\,\dot{x}_{st} + b\,\dot{x}_{st}^2$, therefore the desired damping b_d is set as follows:

$$b_d = \begin{cases} b_{max} & \text{if} \quad y_2^T\,u_2 \leq 0 \\ b_{min} & \text{else} \end{cases} \qquad (21)$$

This control is exactly the same as (19) since:

$$y_2^T\,u_2 = (K\,x_{st} + b\,\dot{x}_{st})\,\dot{x}_{st} = -M_s\,\ddot{x}_s\,\dot{x}_{st} \quad (22)$$

This example and the previous show that the controls C2 and C4 may be simple to be implemented since only a partial knowledge of the system state is required.

4.5 Power-Driven-Damper control

The control (19) may show an oscillating behavior on b_d if the bandwidth β is wide enough and $b_{max}\,\dot{x}_{st}^2 + K\,x_{st}\,\dot{x}_{st} > 0$ and $b_{min}\,\dot{x}_{st}^2 + K\,x_{st}\,\dot{x}_{st} < 0$. This is due to the direct dependence of $\ddot{x}_s$ (or y_2) on the damping b, namely if $\beta \to \infty$ the controlled variable would affect instantaneously the measured variable.

It is clear from (20) (but from (19) it is not!) that the ADD control mimics a sort of sliding mode control of the power whose aim is to steer the power $y_2^T\,u_2$ to zero. From this observation, an alternative strategy is obtained applying the control law C1 with the requirement $W_{2d} = 0$. Matching (7), (17) and (22) we obtain the new Power-Driven-Damper (PDD) control:

$$b_d = \begin{cases} b_{max} & \text{if } K x_{st}\dot{x}_{st} + b_{max}\dot{x}_{st}^2 < 0 \\ b_{min} & \text{if } K x_{st}\dot{x}_{st} + b_{min}\dot{x}_{st}^2 \geq 0 \\ (b_{max} + b_{min})/2 & \text{if } \dot{x}_{st} = 0 \text{ and } x_{st} \neq 0 \\ -K x_{st}/\dot{x}_{st} & \text{otherwise} \end{cases}$$

$$(23)$$

The first two equations in (23) lead to the same behaviour as in (20). The last two equations deal with the problem of the oscillations, the desired damping is indeed set to obtain exactly $W_2 = 0$, namely it equals the equivalent control in a sliding mode sense. This value belongs to the interval $[b_{min}, b_{max}]$. When $\dot{x}_{st} = 0$ the power $W_2 = y_2^T\,u_2$ equals the desired value $W_{2d} = 0$ and the control requirement is satisfied for any damping b. In this case the desired damping b_d is set to the average damping value during transients ($x_{st} \neq 0$) it is set to the minimum in steady state ($x_{st} = \dot{x}_{st} = 0$).

The advantages of the proposed PDD control are clear from the simulation results shown in the next section. The cost is the need for the knowledge of the spring stiffness K and for the exact control of the damping b both required by (23).

4.6 Simulation results

The behaviour of the PDD control is compared to the ADD control in Figs. 2, 3 and 4. Since the ADD control is almost optimal in terms of body acceleration minimization (maximum comfort) the comparison does not take into account the other control strategies less efficient than the ADD. This comparison can be found in (Savaresi *et al.*, 2003) and in (Savaresi *et al.*, 2004). The parameters for the simulations and the comfort evaluation method are the same as in (Savaresi *et al.*, 2004).

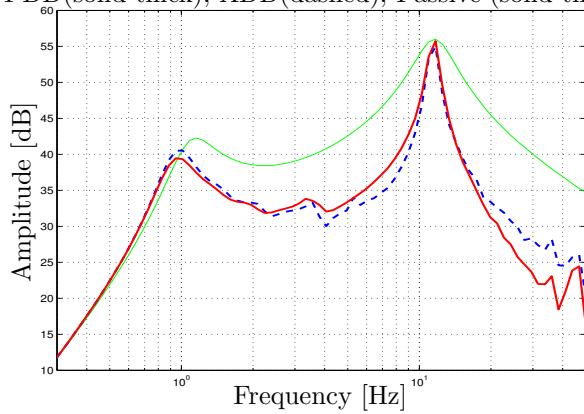PDD(solid-thick), ADD(dashed), Passive (solid-thin).

Fig. 2. Approx. frequency response from the disturbance x_r to the acceleration $\ddot{x}_s$: PDD control , ADD control and passive suspensions.

The comparison of the approximated frequency responses is shown in Fig. 2. The lower is the frequency response, the better is the control algorithm: the ADD is slightly better in the frequency range from 2 to 10Hz, conversely the PDD is slightly better at low and high frequencies.

The time responses shown in Figs. 3 and 4 underlines the advantages of the PDD control in terms of lower jerk both for a sinusoidal and a step road profiles. Concerning the jerk, the improvement is evident. This improvement is not paid in terms of a worsening in the body acceleration, conversely for both the time responses the acceleration behaviors are quite similar: the PDD is slightly better in the step response while the ADD is slightly better for the tone response.

5. CONCLUSIONS

The paper has addressed the problem of controlling port Hamiltonian systems by operating its dissipative terms.

The key idea is to divide the Hamiltonian system into two or more subsystems that are connected by a power preserving connection. To this aim a slight extension of the definition of port Hamiltonian system has been proposed. The control inputs are then chosen to control the stored energy or the power dissipated of a certain subsystems.

The paper presents only some preliminary results, many problems and questions remain open, however the semi-active suspension example has shown that some positive results can be achieved by applying the proposed approach.

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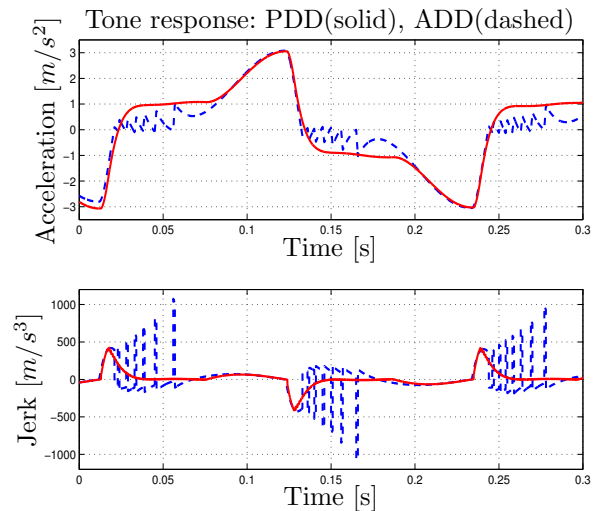
Tone response: PDD(solid), ADD(dashed)

Fig. 3. Time response to a pure 4.5Hz sinusoidal road profile: body acceleration (top) and jerk (bottom) for PDD(solid) and ADD(dashed).
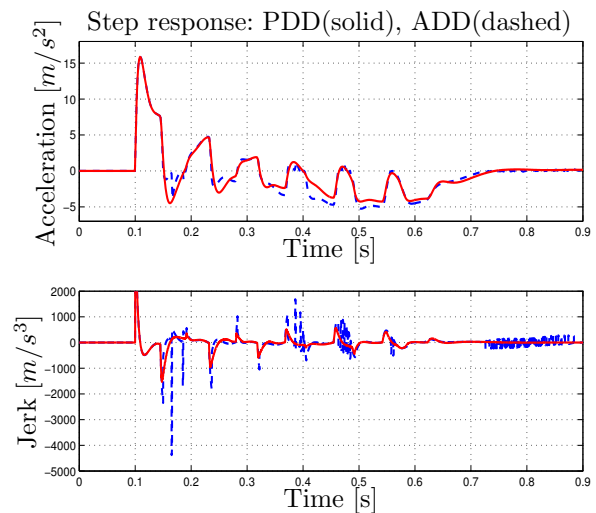
Step response: PDD(solid), ADD(dashed)

Fig. 4. Time response to a step road profile: body acceleration (top) and jerk (bottom) for PDD(solid) and ADD(dashed) controls.

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