Self-Tuning Control Strategy for Antilock Braking Systems

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***Abstract*— One of the main issue of any control strategy for braking systems is to face the many uncertainties due to the strong spread of the system's parameters: road conditions, hydraulic actuators, tire behaviour, etc. Moreover, the need for cheap components limits both the number of sensors and the quality of the actuators.**

This paper proposes a self-tuning control strategy for braking systems. The proposed control strategy is based on two light assumptions: 1) the tire longitudinal force as a function of the tire slip has always a unique minimum; 2) the hydraulic actuators can increase, decrease and hold the braking pressure within a limited delay. Only the measure of the wheel rotational speed and the estimate of the wheel angular acceleration are required. The control strategy is tested by simulation experiments.

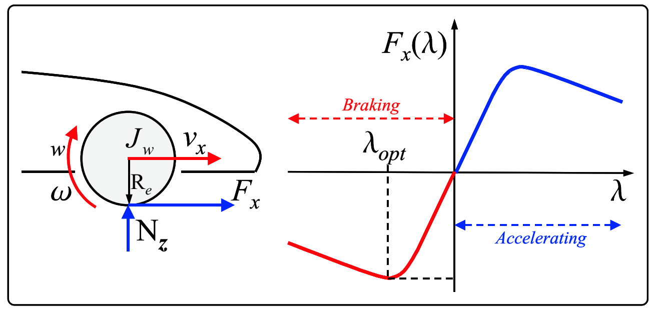

Fig. 1. Basic tire behaviour: slip effects on the longitudinal force.

I. INTRODUCTION

Antilock braking systems (ABS) are now a commonly installed feature in road vehicles. They are designed to stop vehicles as safely and quickly as possible. Safety is achieved by maintaining the steering effectiveness and trying to reduce braking distances over the case where the brakes are controlled by the driver during a "panic stop", see [1].

The ABS control systems are based on the typical tire behaviour described in [2] and briefly shown in Fig. 1. As demonstrated in [3], optimal braking (in terms of minimum traveled distance) occurs when the longitudinal force F_x operates at its minimum value along the force-slip curve. The slip value corresponding to the minimum longitudinal force F_x depends also on the road conditions, vehicle speed, the normal force, the tire temperature, the steering angle, etc. In all cases however the shape of the force-slip curve has a unique minimum for some value of the slip λ. The main issue of the ABS control strategies is to track the optimal slip value λ_{opt} corresponding to the minimum longitudinal force F_x using the smallest number of sensors, using the cheapest hardware and facing the uncertainties due to both the aging of components and the unknown working and environmental conditions. Different control techniques were applied to solve this challenging problem.

Many authors have presented control strategies based on the slip control, see [4], [5], [6], [7] and [8]. Theoretically, the method of slip control is the ideal method. However, two problems arise: the (unknown) optimal slip value must be identified and the vehicle speed must be measured (as in [5], [6]) or estimated in a low cost and reliable way. To overcome these problems, either pressure measurement have been proposed (see [9]) or the braking torque is supposed to be known (see [10], [11], [12]). These solutions lead to very good performances, but do not fit the cost requirements. Moreover, many papers give important theoretical results but do not deal with the dynamics of the actuators.

Currently most commercial ABSs use a look-up tabular approach based on wheel acceleration thresholds, see [1], [13] and [14]. These tables are calibrated through iterative laboratory experiments and engineering field tests. Therefore, these systems are not adaptive and issues such as robustness are not addressed.

The work proposed in this paper shows that it is possible to track the optimal slip value by measuring only the wheel speed and estimating the wheel acceleration. The proposed control strategy can be seen as a minimum seek algorithm based on the phases when the braking pressure (not measured) is kept constant. During these phases the strategy can infer the control action that will increase the braking force. This control strategy is robust with respect to adhesion variations, takes into account the dynamics of the actuators and it is almost hardware independent. The proposed strategy is based on the same assumptions and on the same models usually presented in the literature. Furthermore, differently from the cited papers, the proposed approach takes into account the dynamics of the valves and does not require the measure of the slip, of the hydraulic pressure and of the braking torque.

The paper is organized as follows. The dynamic model of a standard braking system is described in Section II. Based on this model, the basic operating principle of the proposed control is explained in Section III. The control strategy is then described in Section IV and tested by simulations in Section V. Finally some conclusions are drawn in Section VI.

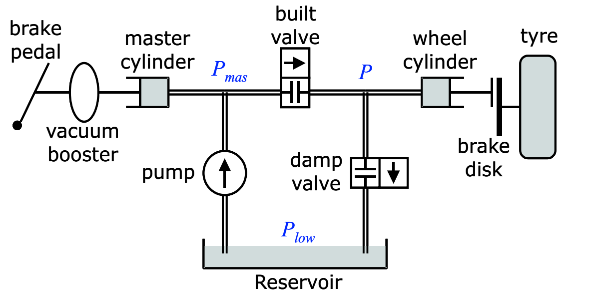

Fig. 2. Schematic of the standard antilock brake system for one wheel.

II. DYNAMIC MODEL OF A BRAKING SYSTEM

The braking system consists of three subsystems: the tires, the vehicle and the electro-hydraulic actuators.

One of the most widely used tire model is based on the Pacejka's "magic formula", see [2]. This is a set of static maps which give the tire forces (longitudinal force F_x, lateral force F_y and self-aligning torque M_z) as a function of the longitudinal slip λ, the slip angle α, the camber angle γ and the vertical load $\mathbf{N}_z$. The static maps are obtained by interpolating experimental data. The longitudinal slip rate λ during braking is defined as:

$$\lambda = \frac{\omega\, R_e - v_x}{v_x} \tag{1}$$

where ω denotes the wheel angular speed, R_e is the rolling radius and v_x is the longitudinal speed of the wheel center in forward direction, see Fig. 1. For a deceleration with constant slip and camber angles (longitudinal braking), v_x is the speed of the vehicle and a qualitative example of the longitudinal force $F_x(\lambda)$ is shown in Fig. 1.

The dynamic behaviour of a wheel during braking is described (see [7],[14], etc) by the differential equation:

$$J_w \dot{\omega} = -K_{brk}\, P - R_e\, F_x(\lambda) \tag{2}$$

where P is the oil pressure in the braking system, K_{brk} denotes the brake gain, $\tau_w = -K_{brk}\, P$ is the braking torque and $F_x(\lambda)$ is the tire longitudinal force.

In this work we consider a simplified model of a single wheel braking vehicle, the dynamics of this quarter vehicle model is described by:

$$M \dot{v}_x = F_x(\lambda) - F_a \tag{3}$$

where M is the mass of the quarter vehicle and F_a is the aerodynamic drag force. During braking $F_x(\lambda)$ is negative. Since both $F_x(\lambda)$ and F_a are limited, it is possible to find the minimum car acceleration A_x^{min} (maximum vehicle deceleration) such that $\dot{v}_x \geq A_x^{min}$ always.

The control strategy (proposed in Section Sec. IV) is almost independent from the hydraulic structure, the only requirement is that the delay of the actuators is limited above by a known value T_d. However, to get the simulation results of Sec. V, the standard (see [9], [14], [15], etc) electro-hydraulic system shown in Fig. 2 has been considered. This system is modeled according to [9] with the addition of a transient to take into account the dynamics of the valves. The duration of the transient is lower than a known value T_d. The master cylinder, the pump and the low pressure reservoir are shared among the 4 wheels, see [1]. Each wheel has two valves: a built valve between the master cylinder and the wheel cylinder and a damp valve between the wheel cylinder and the low pressure reservoir. Both valves are on/off devices and, after a transient, they can only be in two positions: closed or open. This hydraulic structure allows only three control actions:

1) INCREASE: the built valve is open and the damp valve is closed. The braking pressure P increases.
2) HOLD: both the built valve and the damp valve are closed. The braking pressure P, at the end of a transient, can be assumed to be constant.
3) DECREASE: the built valve is closed and the damp valve is open. The braking pressure P decreases.

According to [9], the dynamics of the braking pressure can be modeled by means of a flow through the two valve orifices:

$$C_w \frac{dP}{dt} = A_b\, h(c_b) \sqrt{\frac{2}{\rho}(P_{mas} - P)} - A_d\, h(c_d) \sqrt{\frac{2}{\rho}(P - P_{low})} \tag{4}$$

the coefficients $c_b \in [0,1]$ (built valve) and $c_d \in [0,1]$ (damp valve) are 0 when the corresponding valve is closed, 1 when the valve is completely opened. The dynamics of the actuators is described by:

$$\begin{aligned} \dot{c}_i &= \begin{cases} 0 & \text{if} \quad c_i = 1 \text{ and } u_i \geq 0 \\ 0 & \text{if} \quad c_i = 0 \text{ and } u_i \leq 0 \\ u_i / T_d & \text{else} \end{cases} \\ h(c) &= \begin{cases} 0 & \text{if} \quad c \leq c_0 \\ (c - c_0)/(1 - c_0) & \text{if} \quad c > c_0 \end{cases} \end{aligned} \tag{5}$$

where u_i for $i = b, d$ are the control commands which can take the values 0 or 1. The above relations mean that each valve takes the time T_d to completely open or close. The parameter $c_0 \in [0, 1)$ represents the valve dead zone. If the valve is completely closed ($c_b = 0$ or $c_d = 0$), it takes a time $c_0\, T_d$ to begin to open the valve.

The model of the braking system presented here corresponds to the models described in the literature. Furthermore, the dynamics of the valves is taken into account with a description close to the real hardware.

III. BASIC OPERATING PRINCIPLE

Optimal braking occurs when the longitudinal force F_x operates at its minimum value along the force-slip curve. The proposed ABS control strategy can be seen as a minimum-seek algorithm. Since the force-slip curve has always a minimum, it is first necessary to determine whether the operating point lies in the left or in the right region with respect to this minimum. Then the hydraulic actuators are operated to switch from one region to the other. By this way, a "limit cycle" around the optimal slip value arises and

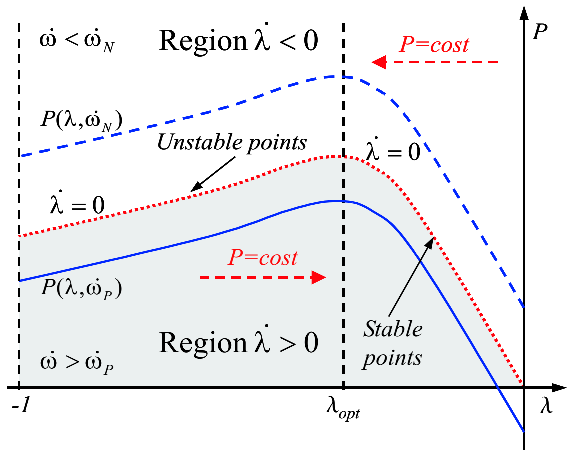

Fig. 3. Qualitative P-λ plot. The dotted line denotes the curve $P_0(\lambda, \dot\omega_0)$ where $\dot\lambda = 0$. The dashed curve $P(\lambda, \dot\omega_N)$ is within the region where $\dot\lambda < 0$, the solid curve $P(\lambda, \dot\omega_P)$ is within the region where $\dot\lambda > 0$.

it guarantees that the longitudinal force F_x varies around its minimum.

Computing the oil pressure in the braking system P from equation (2) we obtain:

$$P(\lambda, \dot\omega) = \frac{-R_e\, F_x(\lambda) - J_w \dot\omega}{K_{brk}} \qquad (6)$$

For a constant value of $\dot\omega = a$ the curve $P(\lambda, a)$ has the same shape of the curve $F_x(\lambda)$. Moreover $a_2 > a_1 \Rightarrow P(\lambda, a_2) < P(\lambda, a_1)$, consequently any acceleration $\dot\omega$ defines an unique curve $P(\lambda, \dot\omega)$ that does not intersect any other $P(\lambda, \dot\omega)$ curve (see Fig. 3), or given P and λ the acceleration $\dot\omega$ is uniquely determined. For any $\dot\omega$ the peak of the curve $P(\lambda, \dot\omega)$ happens for the same value of $\lambda = \lambda_{opt}$. These properties are shown in Fig. 3. This "P-λ plot" is used in the sequel of the paper both to analyze the dynamic behaviour of the tire during braking and to develop the proposed control strategy. The torque-spin diagram presented in [14] is similar to the P-λ plot. This torque-spin plot is introduced in [14] to explain the design of an ABS controller on the basis of an approximated piecewise tire characteristic. The P-λ plot presented here is based on equation (6) and then embeds the (unknown) true tire characteristic.

A wealth of information can be found matching the P-λ plot with the time derivative of the slip rate λ. From (1) the time derivative of the slip λ is:

$$\dot\lambda = R_e \frac{\dot\omega\, v_x - \omega\, \dot v_x}{v_x^2} \qquad (7)$$

where $\dot v_x$ is the longitudinal acceleration A_x of the vehicle. Note that when v_x is reaching zero, the slip λ can vary faster than at high longitudinal speeds. This explains why the worst performance of the ABS controllers happens usually at low speed.

Property 1: *if* $\dot\omega \geq 0$ *then* $\dot\lambda > 0$. Proof: the sign of the slip derivative $\dot\lambda$ is the sign of the term $\dot\omega\, v_x - \omega\, \dot v_x$. During braking $\dot v_x \leq 0$ and ω is limited by the vehicle speed: $v_x \geq R_e\omega \geq 0$. If $\dot\omega \geq 0$ then $\dot\lambda > 0$ and the slip λ increases.

Property 2: *it exists a limited angular acceleration value* $\dot\omega_N$ *such that if* $\dot\omega \leq \dot\omega_N$ *then* $\dot\lambda < 0$. Proof: since the minimum longitudinal acceleration $\dot v_x$ (maximum braking at the best conditions) is limited $A_x^{min} \leq \dot v_x \leq 0$ and during braking $v_x \geq R_e\omega \geq 0$, it exists a limited angular acceleration value such that $\dot\lambda < 0$ is ensured. This acceleration value can be easily found to be $\dot\omega_N = A_x^{min}/R_e$ indeed:

$$\dot\omega < \frac{A_x^{min}}{R_e} = \frac{A_x^{min}\omega}{R_e\omega} \leq \frac{A_x^{min}\omega}{v_x} \leq \frac{\dot v_x \omega}{v_x} \;\Rightarrow\; \dot\lambda < 0$$

Let $\dot\omega_p \geq 0$ be a design parameter. If $\dot\omega \geq \dot\omega_p$ then, thanks to Property 1, $\dot\lambda > 0$. Let $\dot\omega_n \leq \dot\omega_N < 0$ be another design parameter, thanks to Property 2, if $\dot\omega \leq \dot\omega_n$ then $\dot\lambda < 0$. Both $\dot\omega_p$ and $\dot\omega_n$ are "free" parameters that can be tuned to achieve the best possible braking performance.

Somewhere between the two curves $P(\lambda, \dot\omega_N)$ (where $\dot\lambda < 0$) and $P(\lambda, \dot\omega_P)$ (where $\dot\lambda > 0$) lies the curve $P_0(\lambda, \dot\omega_0) \,|\, \dot\lambda = 0$, see Fig. 3. Below [above] the curve $P_0(\lambda, \dot\omega_0)$ the slip increases [decreases] for any value of λ and $\dot\omega$. If the pressure P is kept constant, the points on $P_0(\lambda, \dot\omega_0)$ for $\lambda > \lambda_{opt}$ are stable equilibrium points, while the points on $P_0(\lambda, \dot\omega_0)$ for $\lambda < \lambda_{opt}$ are unstable equilibrium points.

For any oil pressure P, if $\dot\omega \geq \dot\omega_P$ the slip ratio λ is increasing, if $\dot\omega \leq \dot\omega_N$ the slip ratio λ is decreasing. Consequently by measuring the wheel acceleration $\dot\omega$ it is possible to infer some information about the slip ratio. The next step is to find if an operating point of the tire lies in the stable or in the unstable region. Let compute the time derivative of equation (2):

$$J_w\, \ddot\omega = -K_{brk}\, \dot P - \frac{d\, F_x(\lambda)}{d\,\lambda}\, \dot\lambda \qquad (8)$$

The following two properties allow to find where the operating point of the tire is in some working conditions:

Property 3: *if* P *is constant,* $\dot\omega \leq \dot\omega_n$ *and* $\ddot\omega < 0$ *then* $\lambda < \lambda_{opt}$. Proof: since $\dot\omega \leq \dot\omega_n$ from Property 2 follows $\dot\lambda < 0$. The property can now be derived from equation (8) whit $\dot P = 0$.

Property 4: *if* P *is constant,* $\dot\omega \geq \dot\omega_p$ *and* $\ddot\omega < 0$ *then* $\lambda_{opt} < \lambda \leq 0$. Proof: since $\dot\omega \geq \dot\omega_p$ from Property 1 follows $\dot\lambda > 0$. The property can now be derived from equation (8) whit $\dot P = 0$.

IV. SELF-TUNING CONTROL STRATEGY

The proposed control strategy is based on the following assumptions and requirements:

- A.1) During the HOLD phases the oil pressure P and the braking torque $\tau_w = -K_{brk}P$ remain constant. This can be considered true at least for short periods.
- A.2) The wheel angular speed ω is measured. The wheel angular acceleration $\dot\omega$ is measured or estimated.
- A.3) Each control action (HOLD, INCREASE and DECREASE) is ensured within a limited delay. The maximum delay T_d is known.
- A.4) The tire characteristic $F_x(\lambda)$ has a unique minimum.

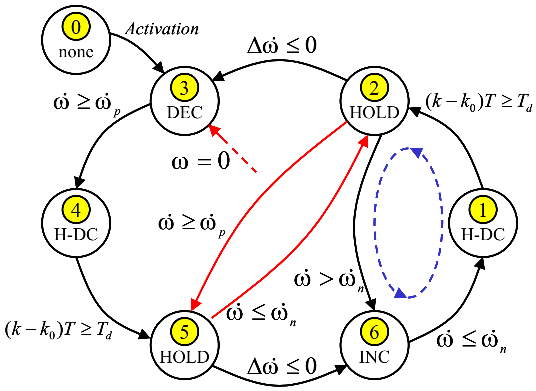

Fig. 4. State chart of the proposed control strategy.

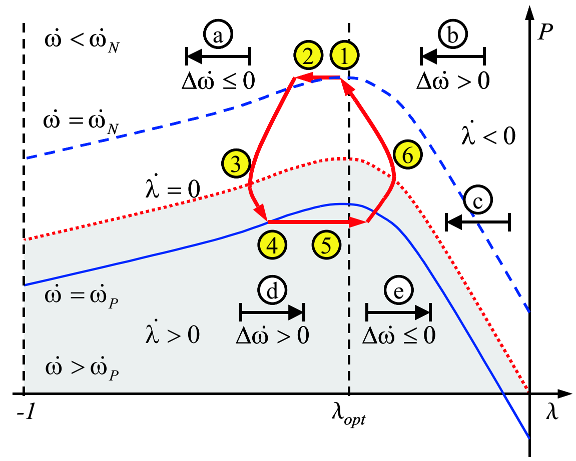

Fig. 5. Basic working cycle represented on the P-λ plot.

Let k denote the current sampling instant and let T be the sampling period of the controller.

Properties 3) and 4) of the previous section requires the second derivative of the wheel speed. This is a problem in a real applications where only the wheel speed is measured. To overcome this problem, the acceleration variation $\Delta\dot{\omega}(k)$ is measured instead of the second derivative $\ddot{\omega}$. A method to get a reliable measure, is to compute $\Delta\dot{\omega}(k)$ by linearly interpolating the acceleration values $\dot{\omega}(i)$ for $i = k - n_h, \ldots, k$ where $n_h \in \mathbb{N}$ is a design parameter that denotes the number of sampling periods that are needed to get a reliable measure of $\Delta\dot{\omega}(k)$. With a small n_h, if the acceleration variation is small the measurement noise will affect the measure. With good low-noise sensors n_h can be small.

The proposed control strategy is based on a 7 state algorithm. The state chart of the algorithm is shown in Fig. 4. The basic working cycle is given by the sequence of states (1)-(2)-(3)-(4)-(5)-(6)-(1), see Fig 5. When the control strategy is active, the control commands can only be HOLD, INCREASE or DECREASE. For some states, a simple initialization assignement is executed once when the algorithm enters the state. The events of each state are checked following the given sequence. The description of the 7 states is the following:

(0) *Control command*: none
Operations:
- if "emergency brake" then next state = (3).

Description:
The ABS control is not active. If a "emergency brake" is detected the ABS control is activated. The activation mode does not affect the behaviour of the proposed control and it is out of the scope of the paper.

(1) *Control command*: HOLD (*Initialization*: $k_0 := k$)
Events:
- if $\omega = 0$ then next state = (3).
- if $(k - k_0)T \ge T_d$ then next state = (2).

Description:
Actuators delay compensation. When $(k - k_0)T \ge T_d$ the actuators delay has been compensated and the HOLD phase has certainly begun.

(2) *Control command*: HOLD (*Initialization*: $k_0 := k$)
Events:
- if $\omega = 0$ then next state = (3).
- if $\dot{\omega} \ge \dot{\omega}_p$ then next state = (5).
- if $(k - k_0) \ge n_h$ and $\Delta\dot{\omega}(k) \le 0$ then next state = (3). Case (a) of Fig 5.
- if $\dot{\omega} > \dot{\omega}_n$ then next state = (6). Case (c) of Fig 5.

Description:
Actuators delay was compensated while in state (1) or (4), the HOLD phase is established and the pressure P can be considered constant. If $(k - k_0) \ge n_h$ the measure of $\Delta\dot{\omega}(k)$ can be considered as reliable.
The three cases (a), (b) and (c) of Fig 5 are now possible. Case (a) corresponds to property 3). In case (b) the HOLD control command is kept since $\lambda > \lambda_{opt}$ and $\dot{\lambda} < 0$. Case (c) is similar to (b), moreover it allows to re-establish an acceleration lower than $\dot{\omega}_N$. By this way a sub-cycle (1)-(2)-(6)-(1) can arise to make λ closer to λ_{opt}.
The second operation is not necessary if the force-slip curve remains constant. It is helpful in case of abrupt changes of the road conditions.

(3) *Control command*: DECREASE
Events:
- if $\dot{\omega} \ge \dot{\omega}_p$ then next state = (4).

Description:
The DECREASE control action is established as soon as the operating point is found to be in the unstable region or when the wheel is locked. By decreasing the brake pressure, the term $R_e F_x(\lambda)$ becomes dominant in equation (2) and the wheel acceleration becomes positive.

(4) *Control command*: HOLD (*Initialization*: $k_0 := k$)
Events:
- if $\omega = 0$ then next state = (3).
- if $(k - k_0)T \ge T_d$ then next state = (5).

Description:
Similar to state (1): actuators delay compensation.

(5) *Control command*: HOLD (*Initialization*: $k_0 := k$)

Events:

- if $\omega = 0$ then next state = (3).
- if $\dot{\omega} \leq \dot{\omega}_n$ then next state = (2).
- if $(k - k_0) \geq n_h$ and $\Delta\dot{\omega}(k) \leq 0$ then next state = (6). Case (e) of Fig 5.

Description:

Similar to state (2). Actuators delay was compensated while in state (4) or (1), therefore the HOLD phase is established and the pressure P can be considered constant. If $(k - k_0) \geq n_h$ the measure of $\Delta\dot{\omega}(k)$ can be considered as reliable.

The two cases (d) and (e) of Fig 5 are possible. Case (e) corresponds to property 4). In case (d) the HOLD control command is kept since $\lambda < \lambda_{opt}$ and $\dot{\lambda} > 0$.

The second operation is not necessary if the force-slip curve remains constant. It is helpful in case of abrupt changes of the road conditions.

(6) *Control command*: INCREASE

Events:

- if $\dot{\omega} \leq \dot{\omega}_n$ then next state = (1).

Description:

The INCREASE control action is established as soon as the operating point is found to be in the stable region. By increasing the brake pressure, the term $K_{brk}\,P$ becomes dominant in equation (2) and the wheel acceleration becomes negative.

V. SIMULATION RESULTS

This section describes the results of two different simulations obtained by changing the valve dynamics and the road conditions. The vehicle and the hydraulic framework are the same for all the simulations. The sampling period is $T = 1$ms. To verify the self-tuning properties, braking on varying road conditions (i.e. dry-wet-dry) have been considered. Fig. 6 shows the two force-slip curves that represent the tire behaviour in the two different road conditions. The transition between the two conditions depends on the traveled distance x. The dynamics of the valves plays an important role for the system performances. Some data about the valves settling time were found in [17]. The system of simulation 1 has average valves ($T_d \leq 20$ms) and sensors ($n_h = 10$), gradual adhesion variations dry-wet-dry. For the simulation 2 the system has slow valves (T_d up to 50ms), noisy sensors ($n_h = 20$), abrupt adhesion variation dry-wet-dry.

<ins>Simulation 1 results</ins>. There are oscillations of the wheel speed and slip due to the valve dynamics, however both the optimal slip and wheel speed are well tracked by the proposed control strategy, as shown in Fig. 7 and in Fig. 8. The braking force is around its maximum, as shown in Fig. 6 and Fig. 9. The performances decay only at low speed (less than 8km/h, last 50cm) when the wheel locks-up. As well known, the controllers based on acceleration thresholds induce oscillations on the braking pressure as shown in Fig. 10.

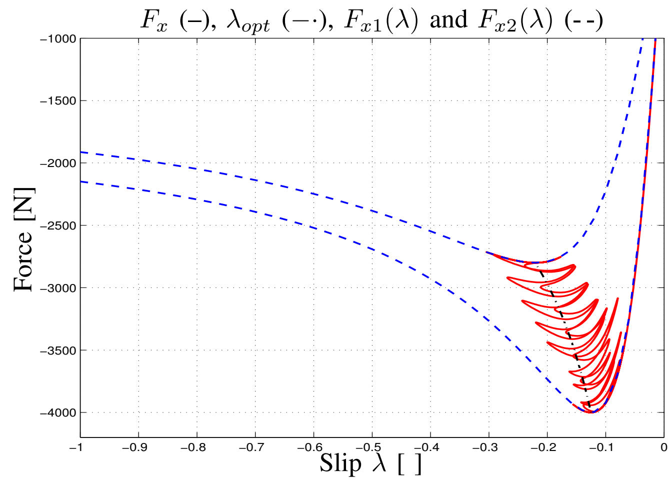

Fig. 6. Simulation 1. Tire longitudinal force F_x (–), tire characteristics $F_{x1}(\lambda)$ and $F_{x2}(\lambda)$ (- -), position of the optimal operating points (–·).

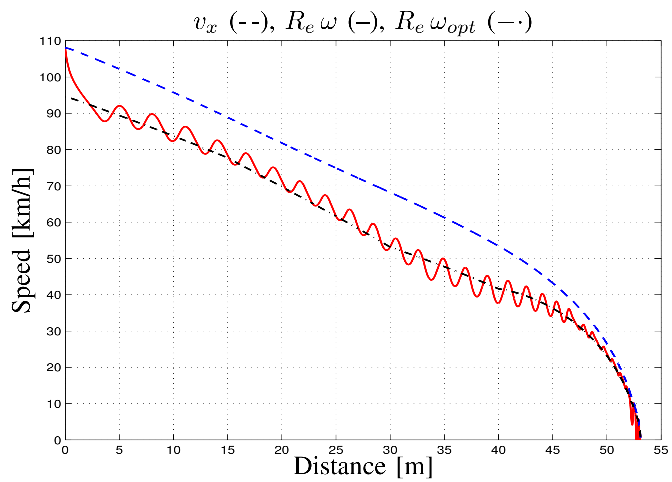

Fig. 7. Simulation 1. Vehicle speed v_x (- -), wheel peripheral speed $R_e\,\omega$ (–) and optimal wheel speed $R_e\,\omega_{opt}$ (–·).

<ins>Simulation 2 results</ins>. The amplitude of the wheel speed and slip oscillations increases with the valves slowness. However these oscillations are still around the optimal slip, as shown in Fig. 11 and in Fig. 12. Consequently the braking force is still around its maximum, as shown in Fig. 13.

VI. CONCLUSIONS

A self-tuning control strategy for antilock braking systems has been proposed. The paper has shown that it is possible to track the optimal slip value by measuring only the wheel speed and estimating the wheel acceleration. The proposed control strategy is robust with respect to adhesion variations, takes into account the dynamics of the actuators and is almost hardware independent. The effectiveness of the control strategy has been tested by simulation experiments.

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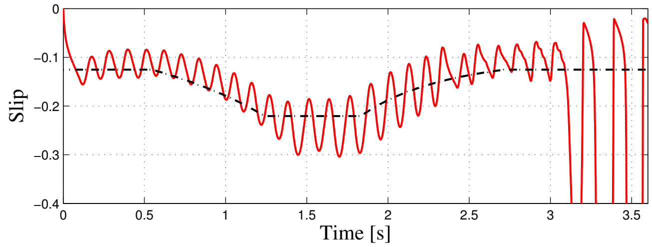

Fig. 8. Simulation 1. Tracking of the optimal slip λ_{opt}: slip λ (-) and optimal slip λ_{opt} (—·).

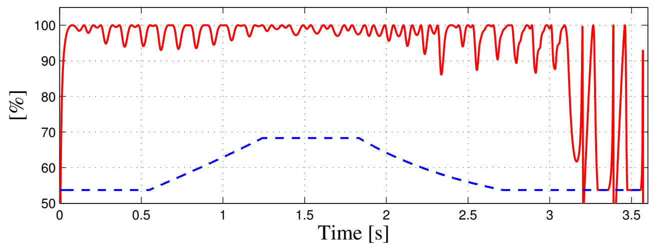

Fig. 9. Simulation 1. Performance evaluation: ratio between the longitudinal force F_x and the maximum achievable force F_{opt}(–). Comparison with the same ratio at wheel lock-up (- -).

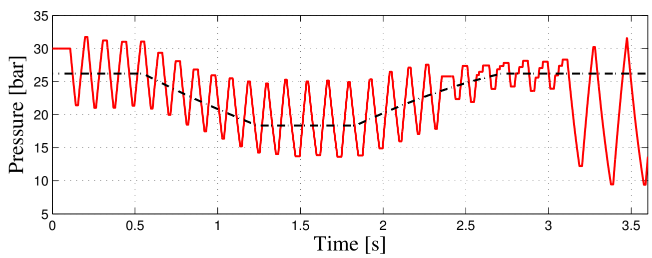

Fig. 10. Simulation 1. Braking pressure P (-) and optimal braking pressure P_{opt} (—·).

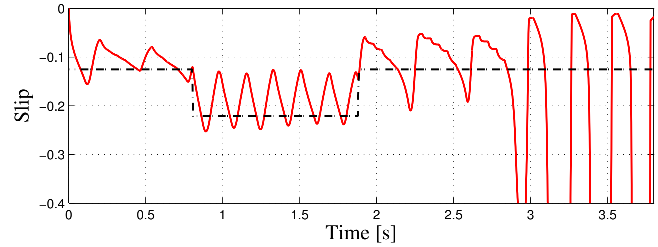

Fig. 11. Simulation 2. Tracking of the optimal slip λ_{opt}: slip λ (-) and optimal slip λ_{opt} (—·).

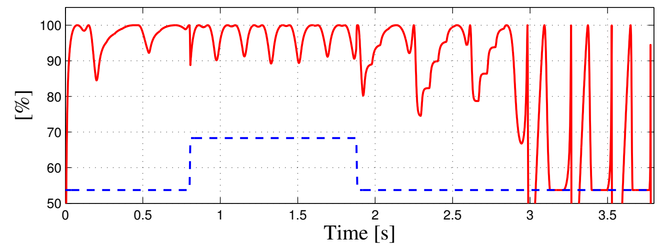

Fig. 12. Simulation 2. Performance evaluation: ratio between the longitudinal force F_x and the maximum achievable force F_{opt}(–). Comparison with the same ratio at wheel lock-up (- -).

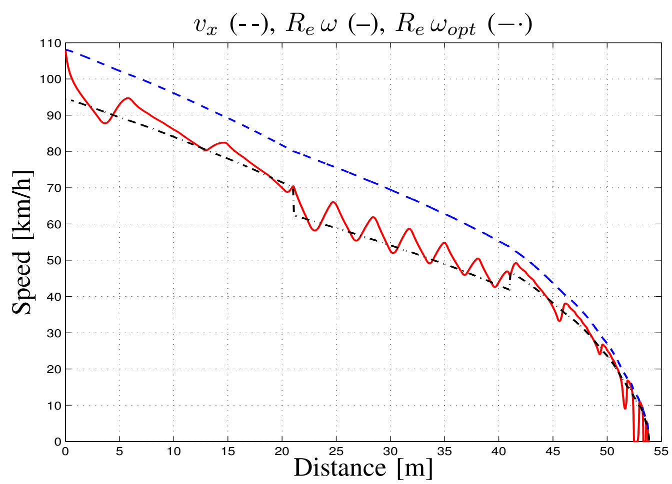

Fig. 13. Simulation 2. Vehicle speed v_x (- -), wheel peripheral speed $R_e\,\omega$ (–) and optimal wheel speed $R_e\,\omega_{opt}$ (—·).

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