

Control of Mechatronic Systems by Dissipative Devices: Application to Semi-Active Vehicle Suspensions

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Abstract—A wide variety of mechatronic systems are controlled by operating dissipative components such as variable resistors, variable dampers, clutches, some electro-valves and more. Facing the limitation that the controlled devices can only dissipate power, the issue is to find a proper control law to satisfy the control requirements.

The mechatronic systems can usually be divided into two or more subsystems that are connected by a power preserving connections. This paper proposes to choose the control inputs to lead the power towards a certain subsystems in order to satisfy the requirements by controlling the energy stored or the power dissipated in that subsystem.

To this aim, the port Hamiltonian framework is used to model the mechatronic systems. A slight extension of the definition of port Hamiltonian system is proposed to allow the description of a larger set of mechatronic systems and to obtain an explicit representation of the energy flowing to a subsystem.

Some of the control laws presented in literature about the control of semi-active suspensions are derived again by following the proposed approach and adding a energetic interpretation.

I. INTRODUCTION

Many mechatronic systems are controlled by operating dissipative components such as variable resistors, variable dampers, clutches, some electro-valves and more. The issue is to find a proper control law that allows to satisfy the control requirements facing the external inputs or disturbances. One of the main issues is the limitation that the energy can only be dissipated by the controlled devices.

The proposed solution is based on a power-oriented modeling of the given mechatronic system. This modeling approach based on power interaction is well known [1] and suitable to mechatronic system [2]. From a mathematical perspective, the *port-controlled Hamiltonian systems* (PHS) (see [3]), are natural candidates to model many physical systems, as shown in the application examples cited in [4]. Basically, PCH are systems defined with respect to a geometric structure capturing the basic interconnection and dissipation laws, and a *Hamiltonian* function given by the total stored energy of the system.

The control of PCH is an interesting research topic and outstanding results have already been obtained. The main results presented so far consider the possibility to operate on the power-ports of a system in order to obtain a controlled closed-loop system that is still a PCH with desired Hamiltonian function, interconnection laws and damping, see [4]

and the references therein. Another approach is the control by interconnection of PCH described in [5]. To the best of our knowledge the problem of controlling a PCH by means of dissipative components has not already been addressed.

The idea proposed in this paper is to divide the mechatronic system into two or more PCH subsystems that are connected by a power preserving connection. If the control requirements are expressed in terms of energy flows or stored energy, the control inputs are chosen to lead the power towards a certain subsystems or to control the stored energy in that subsystem. To this aim, a slight extension of the definition of PCH is proposed to allow the description of a larger set of mechatronic systems and to obtain an explicit representation of the energy flowing to a subsystem. Thanks to the dissipative nature of the controlled devices the passivity properties of the given mechatronic system are preserved.

The semi-active vehicle suspensions (see [6]) are an example of a mechatronic system with a controllable dissipative device. By following the proposed approach, some of the control laws already presented in literature for the semi-active suspensions are derived again and a energetic interpretation is given.

The paper is organized as follows: section II gives a brief introduction on Hamiltonian systems. Section III extends the definition of PCH to include a larger set of mechatronic systems. The proposed control law for the dissipative components are presented in section IV. The application example to semi-active suspensions is described in section V. Finally some conclusions are drawn.

II. A BRIEF INTRODUCTION ON PORT HAMILTONIAN SYSTEMS

The port Hamiltonian framework is a powerful means to model mechatronic and dynamic systems. A brief recall of some definitions given in [3] and in [7] is given herein for reader convenience.

For a standard mechanical system, the *Hamiltonian equations* of motion are $2n$ first-order differential equations:

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + \tau \end{aligned} \tag{1}$$

where $q = (q_1, \dots, q_n)^T$ are generalized configuration coordinates for the system with n degrees of freedom, $\dot{q} = (\dot{q}_1, \dots, \dot{q}_n)^T$ are the generalized velocities, $p = (p_1, \dots, p_n)^T$ is the vector of generalized momenta and the Hamiltonian $H(q, p)$ is the total energy (kinetic $K(q, p)$ and potential $V(q)$) of the system:

$$H(q, p) = K(q, p) + V(q) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + V(q)$$

The $n \times n$ inertia (generalized mass) matrix $M(q)$ is symmetric and positive definite for all q . The generalized momenta are then of the form:

$$p = M(q) \dot{q} \quad (2)$$

System (1) is an example of a *Hamiltonian system* which more generally is given in the following form:

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(q, p) \\ \dot{p} &= -\frac{\partial H}{\partial q}(q, p) + B(q)u \\ y &= B^T(q) \frac{\partial H}{\partial p}(q, p) = B^T(q) \dot{q} \end{aligned} \quad (3)$$

where $B(q) \in \mathbf{R}^{n \times m}$ is the input force matrix, $B(q)u$ denotes the generalized forces resulting from the control inputs $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^m$ are the outputs. The power towards the system is:

$$\frac{dH}{dt}(q(t), p(t)) = u^T(t) y(t)$$

therefore the pair (u, y) represents a *power-port* between the hamiltonian system and the external world.

A major generalization of the Hamiltonian systems described in (3) is called *port-controlled Hamiltonian system* (PCH) and it is the following:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x}(x) + g(x)u, \\ y &= g^T(x) \frac{\partial H}{\partial x}(x) \\ J(x) &= -J^T(x) \\ R(x) &= R^T(x) \geq 0 \end{aligned} \quad (4)$$

One of the key feature of the PCH is the energy perspective in modeling the physical systems. The Hamiltonian $H(x)$ represent the energy stored in the system, the product $y^T u$ has the units of power and has the physical meaning of the power through the port (u, y) , indeed the power balance in (4) is:

$$y^T u = \frac{dH}{dt} + \frac{\partial H^T}{\partial x} R \frac{\partial H}{\partial x} \geq \frac{dH}{dt}$$

namely the power $y^T u$ supplied to the system is partially stored as energy and partially dissipated through R . Several examples of modeling mechatronic systems by Hamiltonian system can be found in the literature, some of them can be found in the references in [4].

III. AN EXTENSION OF THE PORT HAMILTONIAN DEFINITION

Many mechatronic systems are obtained connecting different subsystems by power preserving interconnections. Let (u_1, y_1) and (u_2, y_2) be the power ports of two PCH, the general power preserving interconnection is the following:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 & A \\ -A^T & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad (5)$$

the matrix A can also be time varying and/or state dependent. With the interconnection (5) the power goes from one system to the other without losses:

$$y_1^T u_1 = y_1^T A y_2 = y_2^T A^T y_1 = -y_2^T u_2$$

namely the outgoing energy from one subsystem is exactly the incoming energy to the other.

The PCH in (4) does not consider the possibility of external inputs that directly modify either the dissipation matrix $R(x)$ or the matrix $J(x)$. This problem was partly addressed in [8] where a matrix J depending on external inputs is considered. As previously described, many mechatronic systems have dissipative components whose behaviour depends on an external input. To take into account these phenomena, the following modification of (4) is proposed:

$$\begin{aligned} \dot{x} &= [J(x, v) - R(x, v)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x) \frac{\partial H}{\partial x}(x) \\ J(x, v) &= -J^T(x, v) \\ R(x, v) &= R^T(x, v) \geq 0 \end{aligned} \quad (6)$$

To represent mechatronic systems as a set of PCH connected by power preserving interconnections, the definition (6) is still not enough, as shown in the example of Sec. V. Some components of mechatronic systems may show a direct dissipation between the input u and the output y . A resistor is simplest example. The PCH in (4) cannot consider such behaviour since the dissipation matrix $R(x)$ may vary only due to the state dynamics. To describe mechatronic components that show direct dissipations, this further modification of (6) is proposed:

$$\begin{aligned} \dot{x} &= [J_1(x, v) - R_1(x, v)] \frac{\partial H}{\partial x}(x) + g(x, v)u \\ y &= g^T(x, v) \frac{\partial H}{\partial x}(x) - [J_2(x, v) - R_2(x, v)]u \\ J_i(x, v) &= -J_i^T(x, v) \quad i = 1, 2 \\ R_i(x, v) &= R_i^T(x, v) \geq 0 \quad i = 1, 2 \end{aligned} \quad (7)$$

where v is an external input vector that may also be equal to u . The matrix $J_2(x, v)$ models a direct change of interconnection (example: ideal switches). The matrix $[J_2(x, v) - R_2(x, v)]$ has a similar meaning as the matrix D of the linear systems ($\dot{x} = Ax + Bu, y = Cx + Du$).

The extended definition (7) preserves the basic properties of the PCHs and the energy perspective in modeling the physical systems. The inner product $y^T u$ has still the physical meaning of the power through the port (u, y) and the power balance in (7) is the following:

$$\frac{dH}{dt} = y^T u - \frac{\partial H^T}{\partial x} R_1(x, v) \frac{\partial H}{\partial x} - u^T R_2(x, v) u \quad (8)$$

From (8) it is straightforward to verify that (7) satisfies the energy balance equation (EBE):

$$H(x(t)) - H(x(0)) = \int_0^t y^T(\tau) u(\tau) d\tau - d(t) \quad (9)$$

where $d(t)$ is a nonnegative function that captures the dissipation effects.

IV. CONTROL BY DISSIPATIVE COMPONENTS

Many mechatronic systems are controlled by dissipative components and the inputs u in (7) are not controlled variables, conversely the inputs u often represent disturbances. The semi-active suspension described in the next section is a such example: the input u is the road profile velocity \dot{x}_r . Further examples are the clutches (the torques on the axles are not controlled inputs, only the friction torque is controlled) and some electro-valves (the main external inputs are usually the hydraulic supply pressure and the reservoir pressure). If it is not possible to modify the energy flows by the power port (u, y) , the approach [4] cannot be used. The control requirements can be satisfied by operating the dissipative components only. In the sequel, the external inputs u are considered as non-controllable variables or, similarly, disturbances.

This control problem, to the best of our knowledge, has never been addressed for PCH and a full result is not yet available. This paper proposes to control the dissipative components by taking into account the energy flows between subsystems. The next section shows how some well known control laws can be obtained by using this approach.

The proposed approach is based on two steps:

- 1) translate the control requirements in required energy levels for a subsystem or in required power towards a subsystem.
- 2) operate the dissipative components to obtain the required energy levels or power.

To help the solution of the first step, the mechatronic system is divided into two or more subsystems of the type (6) or (7) that are connected by a power preserving connection of the type (5). By this way the power toward and the energy stored in each subsystem can be easily computed. The correspondence between control requirements and energy levels or power is the target of future research and it is not addressed in this paper.

Concerning the second step, the control inputs v are chosen to control the power towards a certain subsystems

in order to control the energy stored or the power dissipated in that subsystem. Four control laws for the input v are proposed. The control, to a desired value W_d , of the power to a subsystem is the target of the control laws C1 and C2. The control laws C3 and C4 are based on a desired energy level H_d for the subsystem. To simplify the notation, let $D(v, x)$ denote the dissipated power:

$$\begin{aligned} D(v, x) &= \frac{\partial H^T}{\partial x} R_1(x, v) \frac{\partial H}{\partial x} + u^T R_2(x, v) u \geq 0 \\ \dot{H} &= y^T u - D(v, x) \end{aligned} \quad (10)$$

C1) Let W_d be the desired value of the power toward a subsystem of the type (6) or (7). From the power balance (10) the desired power W_d or its closest possible values are obtained by the following control law:

$$v : \begin{cases} \max(\dot{H} + D(v, x)) & \text{if } \max(\dot{H} + D(v, x)) < W_d \\ \min(\dot{H} + D(v, x)) & \text{if } \min(\dot{H} + D(v, x)) > W_d \\ W_d = \dot{H} + D(v, x) & \text{otherwise} \end{cases} \quad (11)$$

The first (or the second) condition means that the desired power W_d cannot be obtained and then one of the values of v that gives the smallest difference is chosen. The desired value W_d is only obtained in the third case. Unfortunately to solve (11) (especially for the third case) the exact knowledge of the system $(x, H(x), R_1(x, v)$ and $R_2(x, v)$ must be exactly known) and of the power through the port (u, y) are required.

C2) When it is required to minimize or maximize the power W_d the control law (11) can be simplified and the resulting new control law may be much easier to be implemented. The minimization (maximization) of W_d can be obtained by substituting in (11) $W_d = -\infty$ ($W_d = \infty$). If $W_d = \pm\infty$ only some conditions of (11) are possible and the control law simplifies as follows:

$$v : \begin{cases} \max(\dot{H} + D(v, x)) & \text{if } W_d = +\infty \\ \min(\dot{H} + D(v, x)) & \text{if } W_d = -\infty \end{cases} \quad (12)$$

As shown in the next section, this control law may be much simpler to be implemented since only the maximization (or minimization) of $\dot{H}(x) + D(v, x)$ with respect to v is required.

C3) Let H_d be the desired level of energy for the considered subsystem. Let $f(z)$ be a real function of the real variable z such that $f(z)z > 0$ if $z \neq 0$. The desired energy level H_d can be obtained by the following control law:

$$\dot{H}_d = -f(H(x) - H_d) \quad (13)$$

where \dot{H}_d is the desired value for the time-derivative $\dot{H}(x)$. From (8) the time derivative \dot{H}_d can be obtained choosing the control v as follows:

$$v : \begin{cases} \max(y^T u - D(v, x)) & \text{if } \max(y^T u - D(v, x)) < \dot{H}_d \\ \min(y^T u - D(v, x)) & \text{if } \min(y^T u - D(v, x)) > \dot{H}_d \\ \dot{H}_d = y^T u - D(v, x) & \text{otherwise} \end{cases} \quad (14)$$

The first (or the second) condition means that the desired \dot{H}_d cannot be obtained and then one of the values of v that gives the smallest difference is chosen. The desired value \dot{H}_d is only obtained in the third case. Unfortunately to solve (13) the exact knowledge of the system (x , $H(x)$, $R_1(x, v)$ and $R_2(x, v)$ must be exactly known) and of the power through the port (u, y) are required.

C4) A control law that requires less knowledge of the system than C3 is obtained starting from (13) and (14) and choosing the maximum or the minimum possible values for \dot{H}_d . The resulting control law is the following:

$$v : \begin{cases} \max(y^T u - D(v, x)) & \text{if } H(x) < H_d \\ \min(y^T u - D(v, x)) & \text{if } H(x) \geq H_d \end{cases} \quad (15)$$

As shown in the next section, this control law may be much simpler to be implemented since only the maximization (or minimization) of $y^T u - D(v, x)$ with respect to v is required.

Remark 1. As shown in equations (11) and (14) it is not ensured that the control requirements can always be satisfied by operating on the control input v . This is mainly due to the inputs u that are not controlled variables and that may assume any value while the term $D(v, x)$ may be limited.

Remark 2. In the more general case the input v is a vector, therefore equations (11), (12) (14) and (15) may have more solutions that have different components of v . If a particular structure for the matrices R_1 and R_2 is not given, it is not possible to define a criterium for the choice of the best solution.

Remark 3. The considered mechatronic systems are described by PCHs of the type (6) or (7) interconnected by (5) therefore the energy balance equation (9) is always satisfied and the subsystems are all passive for any value of the input v . Therefore, thanks to the dissipative nature of the controlled devices the passivity properties of the given mechatronic system are preserved for any choice of the control law.

V. CONTROL OF SEMI-ACTIVE SUSPENSIONS

The semi-active suspensions are a typical example of a mechatronic system controlled by a dissipative component. A semi active suspension system is shown in Fig. 1 regarding a quarter-car model. The damping b of the shock absorber is controlled by an electro-valve. The typical control problem is to choose the value of the desired damping b_d in order to maximize the comfort for the passengers. The ideal solution

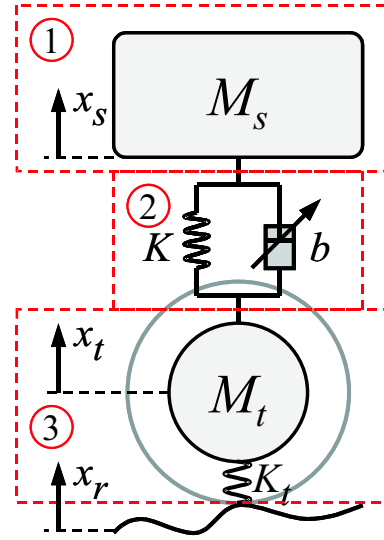


Fig. 1. Graphical representation of the quarter-car model with semi-active suspension with variable damper b .

were to obtain a body (sprung mass) speed and acceleration as close as possible to zero to minimize the movements and the forces perceived by the passengers. A detailed description of the semi-active suspensions can be found in [6] and in references therein. This section shows how some control laws already known for the semi-active suspensions can be derived again by means of the proposed approach. Other examples are also available (clutches, electro-valves,...), however the semi-active suspensions example is one of the most complete and intuitive.

The variables shown in Fig. 1 have the following meanings: M_s denote the quarter-car body mass, M_t is the total unsprung mass (tire, wheel, brakes, suspension links,...), b and b_d are the real and the desired damping coefficients of the shock-absorber, K and K_t are the stiffness of the suspension spring and of the tire, respectively. Finally x_s , x_t and x_r are the vertical position of the body mass, of the tire and of the road profile, respectively. The PCH model of the system shown in Fig. 1 is the following:

$$H = \frac{1}{2} M_s \dot{x}_s^2 + \frac{1}{2} M_t \dot{x}_t^2 + \frac{1}{2} K x_{st}^2 + \frac{1}{2} K_t x_{tr}^2$$

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_t \\ \dot{x}_{st} \\ \dot{x}_{tr} \end{bmatrix} = \begin{bmatrix} \frac{-b}{M_s^2} & \frac{b}{M_s M_t} & \frac{-1}{M_s} & 0 \\ \frac{b}{M_s M_t} & \frac{-b}{M_t^2} & \frac{1}{M_t} & \frac{-1}{M_t} \\ \frac{1}{M_s} & \frac{-1}{M_t} & 0 & 0 \\ 0 & \frac{1}{M_t} & 0 & 0 \end{bmatrix} \begin{bmatrix} M_s \dot{x}_s \\ M_t \dot{x}_t \\ K x_{st} \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \dot{x}_r$$

$$y = [0 \ 0 \ 0 \ -1] [M_s \dot{x}_s \ M_t \dot{x}_t \ K x_{st} \ K_t x_{tr}]^T \quad (16)$$

The gravitational force has been compensated by the springs pre-load and it does not compare in the equations. The state variables $x_{st} = x_s - x_t$ and $x_{tr} = x_t - x_r$ represent, respectively, the deformations of the spring and of the tire with respect to the equilibrium length.

The variable dissipation b depends on the actuator dynamics. The simplest actuator is usually described by a first order linear dynamics with saturation of b between $b_{min} > 0$ and $b_{max} > b_{min}$. Let b_d be the desired damping, let $\beta > 0$ be the bandwidth of the actuator, the simplified actuator dynamics is the following:

$$\dot{b} = \begin{cases} 0 & \text{if } b \geq b_{max} \text{ and } b_d \geq b_{max} \\ 0 & \text{if } b = b_{min} \text{ and } b_d \leq b_{min} \\ \beta(b_d - b) & \text{else} \end{cases}$$

Remark 4. For the sake of clarity the described suspension system is linear as in [6]. For a real suspension system both K and b are nonlinear functions of some state variables. However the results presented in the following section hold also in the nonlinear case and for any (regular) actuator dynamic behaviour.

A. Partition of the PCH

The semi active suspension system can be partitioned in the following three connected PCHs (see also the dashed boxes of Fig. 1):

1) Subsystem 1, sprung mass PCH:

$$\begin{aligned} H_1 &= \frac{1}{2} M_s \dot{x}_s^2 \\ \ddot{x}_s &= [0] M_s \dot{x}_s + \left[\frac{1}{M_s} \right] u_1 \\ y_1 &= \left[\frac{1}{M_s} \right] \frac{\partial H_1}{\partial \dot{x}_s} = \dot{x}_s \end{aligned} \quad (17)$$

2) Subsystem 2, spring-damper PCH:

$$\begin{aligned} H_2 &= \frac{1}{2} K x_{st}^2 \\ \dot{x}_{st} &= [0] K x_{st} + [1 \quad -1] \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix} \\ \begin{bmatrix} y_{2,1} \\ y_{2,2} \end{bmatrix} &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} K x_{st} + \begin{bmatrix} b & -b \\ -b & b \end{bmatrix} \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix} \end{aligned} \quad (18)$$

3) Subsystem 3, wheel and tire PCH:

$$\begin{aligned} H_3 &= \frac{1}{2} M_t \dot{x}_t^2 + \frac{1}{2} K_t x_{tr}^2 \\ \begin{bmatrix} \dot{x}_t \\ \dot{x}_{tr} \end{bmatrix} &= \begin{bmatrix} 0 & -\frac{1}{M_t} \\ \frac{1}{M_t} & 0 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} + \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_{3,1} \\ \dot{x}_r \end{bmatrix} \\ \begin{bmatrix} y_{3,1} \\ y_{3,2} \end{bmatrix} &= \begin{bmatrix} \frac{1}{M_t} & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} M_t \dot{x}_t \\ K_t x_{tr} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ -K_t x_{tr} \end{bmatrix} \end{aligned} \quad (19)$$

The three subsystems are connected in the following power-preserving way:

$$\begin{aligned} \begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,1} \\ y_1 \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,1} \\ u_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_s \\ -K x_{st} - b \dot{x}_{st} \end{bmatrix} \\ \begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_{2,2} \\ y_{3,1} \end{bmatrix} \rightarrow \begin{bmatrix} u_{2,2} \\ u_{3,1} \end{bmatrix} = \begin{bmatrix} \dot{x}_t \\ K x_{st} + b \dot{x}_{st} \end{bmatrix} \end{aligned} \quad (20)$$

In the sequel two of the control law proposed for the semi-active suspensions are derived again by following the proposed approach. Although the passive suspensions are not controllable, their behavior from a power/energy perspective is analyzed to get some insight about the proposed approach.

B. Passive suspensions

Let consider the spring-damper subsystem 2, the stored energy is:

$$H_2 = \frac{1}{2} K x_{st}^2 \geq 0$$

and the power $y_2^T u_2 = \dot{H}_2 + D_2(x, v)$ results:

$$\begin{aligned} y_2^T u_2 &= \dot{H}_2 + D_2(v, x) \\ &= K x_{st} \dot{x}_{st} + [u_{2,1} \quad u_{2,2}] \begin{bmatrix} b & -b \\ -b & b \end{bmatrix} \begin{bmatrix} u_{2,1} \\ u_{2,2} \end{bmatrix} \\ &= K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2 \end{aligned} \quad (21)$$

If the requirement were to dissipate as much power as possible from the external world, the control law C2 with $W_{2d} = +\infty$ should be applied. From (12) and (21):

$$b : \max(K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2) \rightarrow b_d = \text{const} = b_{max}$$

therefore the damping should be constant at its maximum value and a passive suspension is enough to meet the requirement. However it is well known that this solution is not optimal both for performance and for comfort.

C. Classic two-state sky-hook control strategy

The target of the sky-hook control is to keep the body vertical speed \dot{x}_s and acceleration \ddot{x}_s as close as possible to zero facing the road profile \dot{x}_r . The classic two-state ‘‘sky-hook’’ control law given in the literature, see [6], is the following:

$$b_d = \begin{cases} b_{max} & \text{if } \dot{x}_{st} \dot{x}_s \geq 0 \\ b_{min} & \text{else} \end{cases} \quad (22)$$

This control law can be obtained in an alternative way by means of the method presented previously following energetic considerations. Consider the subsystem 1 described by (17), the kinetic energy of the body is $H_1(\dot{x}_s)$ and it is always positive. Let the desired energy level H_{1d} be set to zero, this means zero vertical speed of the body. By applying the control law C4, only the second condition of (15) is possible. Since for the subsystem 1 $D_1(x, v) = 0$, the power $y_1^T u - D(x, v)$ results:

$$y_1^T u_1 - D_1(x, v) = \dot{x}_s (-b \dot{x}_{st} - K x_{st}) = -b \dot{x}_s \dot{x}_{st} - K x_{st} \dot{x}_s$$

To minimize $y_1^T u_1 - D_1(x, v)$ as requested in (12) it is only possible to minimize $-b \dot{x}_s \dot{x}_{st}$, this is obtained exactly by requiring the damping b_d as in the control law (22).

This example shows that the control law C4 may be simple to be implemented since only a partial knowledge of the system state is required.

D. Acceleration based two-state sky-hook control strategy

This strategy is proposed in [6] and it is based on the body vertical acceleration:

$$b_d = \begin{cases} b_{max} & \text{if } \dot{x}_s \dot{x}_{st} \geq 0 \\ b_{min} & \text{else} \end{cases} \quad (23)$$

This control law can be obtained in an alternative way by means of the control law C2 previously presented. Consider the subsystem 2 described by (18). Let the desired power W_{2d} be the following:

$$W_{2d} = \begin{cases} +\infty & \text{if } y_2^T u_2 \leq 0 \\ -\infty & \text{if } y_2^T u_2 > 0 \end{cases} \quad (24)$$

The incoming power is $y_2^T u_2$ and $W_d = y_{2d}^T u_{2d}$ consequently the requirements are to keep the incoming power as close as possible to zero: if $y_2^T u_2 > 0$ ($y_2^T u_2 < 0$) the requested power W_{2d} is the lowest (highest) possible. This control law mimics a sort of sliding mode control of the power. Using (8), the power balance equation is:

$$y_2^T u_2 = \dot{H}_2 + D_2(v, x) = K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2 \quad (25)$$

According to (12), (24) and (25), the damping b must be chosen to maximize (or minimize) $K x_{st} \dot{x}_{st} + b \dot{x}_{st}^2$, therefore the desired damping b_d is set as follows:

$$b_d = \begin{cases} b_{max} & \text{if } y_2^T u_2 \leq 0 \\ b_{min} & \text{else} \end{cases} \quad (26)$$

This control law is exactly the same as (23) since:

$$y_2^T u_2 = (K x_{st} + b \dot{x}_{st}) \dot{x}_{st} = -M_s \ddot{x}_s \dot{x}_{st}$$

This example shows that the control law C2 may be simple to be implemented since only a partial knowledge of the system state is required.

Remark 5. The control law (23) may show an oscillating behavior on b_d if the bandwidth β is wide enough and when $b_{max} \dot{x}_{st}^2 + K x_{st} \dot{x}_{st} > 0$ and $b_{min} \dot{x}_{st}^2 + K x_{st} \dot{x}_{st} < 0$. This is due to the direct dependence of \dot{x}_s (or y_2) on the damping b , namely if $\beta \rightarrow \infty$ the controlled variable would affect instantaneously the measured variable.

The choice of the requirement (26) is then a limit case for the control law C2.

VI. CONCLUSIONS

The paper has addressed the problem of controlling mechatronic systems, described by interconnected port Hamiltonian systems, by operating dissipative components. To this aim a slight extension of the definition of port Hamiltonian system has been proposed to allow the description of a larger set of mechatronic systems.

The proposed idea is to divide the mechatronic system into two or more subsystems that are connected by a power preserving connection. The control inputs are then chosen to control the stored energy or the power dissipated of a certain subsystems.

The semi-active suspension has been considered as a basic example of application of the proposed approach.

The paper presents only some preliminary results, many problems and questions remain open, among them translating the control requirements into a desired energy or power and choosing the best control law are open issues for further research.

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